DOMAIN AND RANGE OF A FUNCTION

- Want some basic practice with functions first?
  Introduction to Functions
  Introduction to Function Notation
  More Practice with Function Notation


(more mathematical cats)

The ***domain*** of a function is the set of all its allowable inputs.
The ***domain convention*** states that if the domain of a function is not specified, then it is assumed to be the set of all real numbers for which the function is defined.

The ***range*** of a function is the set of all its outputs, as the inputs vary through the entire domain.

The domain of a function f is denoted by $\text{dom}(f)$.
The range of a function f is denoted by $\text{ran}(f)$.

Since the domain and range are ***sets***, correct set notation must be used when reporting them.
It may be helpful to review interval and list notation.
Remember that the symbol $\mathbb{R}$ denotes the set of real numbers.

The domain of a function is usually quite easily determined from the formula for the function.
Numbers that cause division by zero must be excluded from the domain.
Anything inside an even root (square root, fourth root, etc.) must be greater than or equal to zero.

The range of a function is usually more difficult to determine from a formula.
Often, it is much easier to get the range from a graph of the function (which is the topic of a future section).
In this exercise, you are only asked to find the range for very simple functions.

EXAMPLES:

Question:
What is the domain of the function f defined by $f(x) = \sqrt{x+2}$?
Solution:
Since any number inside a square root must be nonnegative, we must have: $x + 2 \geq 0$
Subtracting 2 from both sides: $x \geq -2$
Thus, the domain is the set of all real numbers that is greater than or equal to -2.
Using interval notation, and writing a complete mathematical sentence to report the answer:
$\text{dom}(f) = [-2, \infty)$

Question:
What is the domain of the function f defined by $f(x) = 3$?

Solution:
This is a constant function!
No matter what the input is, the output is the number 3.
For example:

- $f(0) = 3$
- $f(-2.79) = 3$
- $f(\pi) = 3$

All real numbers can be inputs.
Thus, using interval notation, $\text{dom}(f) = (-\infty, \infty)$.
Alternatively, you could write $\text{dom}(f) = \mathbb{R}$.

Question:
What is the range of the function f defined by $f(x) = 3$?

Solution:
No matter what the input is, the output is the number 3.
Thus, the range contains only one number, 3.
Using list notation, $\text{ran}(f) = \{3\}$.

Question:
What is the range of the function g defined by $g(x) = \sqrt{x}$?

Solution:
The outputs from square roots are always nonnegative (greater than or equal to zero).
And, you can get ***any*** nonnegative number as an output, just by taking its square as an input!

For example, suppose you want to get the output 1.528 from the function g.
Just use 1.528^2 as the input: $g(1.528^2) = \sqrt{1.528^2} = 1.528$

Using interval notation, $\text{ran}(g) = [0, \infty)$.

Question:

What is the domain of the function f defined by $f(x) = \dfrac{-2}{3x+5}$?

Solution:

The denominator cannot equal zero; that's the only restriction.
Thus, we have to exclude any value of x for which $3x + 5 = 0$:

$3x + 5 = 0$
$3x = -5$
$x = -\frac{5}{3}$

So, the domain is the set of all real numbers ***except*** $-\frac{5}{3}$.
That is, we want all real numbers less than $-\frac{5}{3}$, put together with all real numbers greater than $-\frac{5}{3}$.
The 'union' symbol, $\cup$, is used to 'put sets together'.
Thus, $\text{dom}(f) = (-\infty, -\frac{5}{3}) \cup (-\frac{5}{3}, \infty)$.

This is the last exercise in Algebra I.
(If you went through the entire course,
congratulations are definitely in order!)
From here, you might want to move on to
Geometry or Algebra II.